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Mathematical models of hydromechanics of multiphase flow with varying mass

Matematyczne modele hydromechaniki przepływu wielofazowego o zmiennej masie

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ABSTRACT: The paper discusses the mathematical model of hydromechanics of multiphase flows with varying mass. A multiphase flow is considered a continuum consisting of a set of a large number of different groups of particles. The derivation of motion equations and similarity criteria are given taking into account both the externally attached (or detached) mass and phase transitions within the medium. The equations of mass, momentum and energy transfer for individual phases and the medium as a whole are derived based on fundamental conservation laws. It was demonstrated that in the absence of sources (or flow-offs) of mass, momentum and energy, the known equations of single- and multi-phase flow hydromechanics follow as a special case from the obtained systems of motion equations and similarity criteria. The obtained motion equations are valid for the description of an ingredient of mixture and the medium as a whole, regardless of their physical and mechanical properties. Thermodynamic and rheological state equations, as well as expressions for heat flow, interfacial mass forces phase transitions, and heat exchange between phases can be used to close them. The implemented models make it possible to simulate both the stationary distribution of parameters along the wellbore during production and non-stationary processes that occur, for example, when the pump shaft speed changes during oil production. The developed approaches were implemented in the DataFlow software tool for analysis of the hydrodynamics of multiphase hydrocarbon flows, taking into account heat exchange with the rocks surrounding the well, and phase transitions in the fluid. Using the software package, test calculations were carried out to demonstrate the performance of the proposed and implemented models.

Key words: multiphase flow, varying mass, continuum, phase transition, viscous fluid, tension, pressure.

STRESZCZENIE: W artykule omówiono model matematyczny hydromechaniki przepływów wielofazowych o zmiennej masie. Przepływ wielofazowy jest traktowany jako kontinuum składające się ze zbioru dużej liczby różnych grup cząstek. Wprowadzone równania ruchu i kryteria podobieństwa są podane z uwzględnieniem zarówno zewnętrznej dołączonej (lub odłączonej) masy, jak i przejść fazowych wewnątrz ośrodka. Równania transferu masy, pędu i energii dla poszczególnych faz i ośrodka jako całości otrzymano przy użyciu podstawowych praw zachowania. Wykazano, że w przypadku braku źródeł (lub wypływów) masy, pędu i energii, znane równania hydromechaniki przepływu jedno- i wielofazowego wynikają jako szczególny przypadek z otrzymanych układów równań ruchu i kryteriów podobieństwa. Uzyskane równania ruchu mają zastosowanie do opisu składnika mieszaniny i medium jako całości, niezależnie od ich właściwości fizycznych i mechanicznych. Do ich rozwiązania można wykorzystać termodynamiczne i reologiczne równania stanu, a także wyrażenia dotyczące przepływu ciepła, siły międzyfazowych masy, przejść fazowych i wymiany ciepła między fazami. Wdrożone modele umożliwiają symulację zarówno stacjonarnego rozkładu parametrów wzdłuż odwiertu podczas wydobycia, jak i procesów niestacjonarnych, które zachodzą na przykład podczas zmiany prędkości wału pompy w trakcie wydobycia ropy. Opracowane podejścia zostały zaimplementowane w oprogramowaniu DataFlow do analizy hydrodynamiki wielofazowych przepływów węglowodorów, z uwzględnieniem wymiany ciepła ze skałami otaczającymi odwiert oraz przejść fazowych w płynie. Za pomocą pakietu oprogramowania przeprowadzono obliczenia testowe w celu wykazania wydajności proponowanych i wdrożonych modeli.

Słowa kluczowe: przepływ wielofazowy, zmienna masa, kontinuum, przejście fazowe, lepka ciecz, napięcie, ciśnienie.

Introduction

The range of problems related to the multiphase flow hydromechanics is extensive and has been developing intensively in recent years. This is due to important practical applications in various scientific and technical fields, such as energy, petrochemistry, drilling, mechanical engineering, chemical technology, agricultural engineering, etc. Studies of multiphase flow

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hydromechanics are progressing in several directions, each having its own specifications and peculiarities, both in terms of theoretical description and experimental study (Deich and Filippov 1981; Timofeeva, 2014).

An analysis of known papers has shown that currently, the basic equations of multiphase flow are established in the absence of external sources (flow-offs) of mass, momentum, and energy (i.e., with the constancy of mass) of the mixture flow.

However, in many important practical works, the total mass of the mixture flow undergoes significant changes (mass variability is understood here otherwise than in the theory of relativity and is a consequence of the changes in the composition of particles forming the mixture) due to the addition (or detachment) of a new mass (through permeable flow loops). Such flows are common in the following areas: oil, gas and water distribution (or gathering) systems (pipes or channels); injection or suction in boundary layer control; injector and separation systems of dissipating and drainpipes; diverging and converging ducts; manifold heat exchangers; continuous flow setting tanks and tanks with hydromechanical (water jet) cleaning, etc. What these problems have in common is that the flow movement in their flow part involves a change in mass (i.e., with the addition or detachment of mass along the path). These considerations are essential for a comprehensive approach to the problems of multiphase flow hydromechanics. The movement of a multiphase (two-phase, inhomogeneous) mixture in the vast majority of natural and technical systems is turbulent, making its study a crucial practical task.

When mathematically describing the motion of a multiphase turbulent flow, stylized laws of mechanics are used. The methods of operational analysis proposed by different researchers, at various times, for the mathematical description of the movement of a multiphase (two-phase) flow have different degrees of approximation and specific limited areas of application.

One of the main challenges in formulating differential equations for the motion of a turbulent multiphase (two-phase, suspension-carrying) flow is the presence of surfaces of weak and strong discontinuities in a turbulent flow of a mixture, where characteristics of the flow change chaotically and disorderly in in time and at each point in space, both in magnitude and in direction. Therefore, the actual values of velocity and pressure of a multiphase flow, strictly speaking, cannot be considered continuous functions of space and time coordinates throughout the entire space occupied by the mixture.

Continuous functions of space and time coordinates in a turbulent multiphase or two-phase flow are considered to be the averaged values of velocities and pressures, for both the liquid and suspended (solid) phases, to which the laws of continuum mechanics are already applicable (the concept of continuity has certain limitations and requires compliance with a group of conditions even for single-phase flow) (Karaushev, 2016).

As is known, mixing (diffusion) is one of the main properties of turbulence. Reasoning in general terms, we can say that a mass of liquid moving at an average speed diffuses (fluctuates) with a pulsating speed. If we temporarily ignore the multiphase (two-phase) flow and consider the flow without the presence of suspended particles, it is easy to notice significant differences between molecular and turbulent diffusion. In the case of molecular diffusion, the medium consists of discrete particles, whereas in turbulent diffusion, the medium is quasi-continuous.

An analogy can be drawn between molecular and turbulent diffusion in a multiphase (two-phase, suspended) flow, where the medium is discrete with respect to the suspended (carrying) phase. However, in a single-phase flow, when considering issues of turbulent diffusion, the discreteness of the medium is assumed in relation to individual particles of liquid volumes (for example, in relation to vortices), as if they were composed of molecules of a different kind, which retain their original properties throughout the entire period of the movement process. This implies that the spatial distribution of particles changes (Anderson et al., 2012).

In a mixture of multiphase (two-phase, suspension-carrying) flows, discreteness takes on a more pronounced form than in a single-phase flow. This is due to the presence of practically non-deformable solid particles of another material in such flows of a deformable medium (in the presence of an aerated flow, in a less deformable medium there are more deformable particles), which leaves its mark on the continuity of the mixture as a whole (Harrer et al., 2021).

In the mathematical sense, averaging allows us to transition from fields of vector and scalar quantities that change abruptly in time and space to fields of the same quantities that change smoothly in time and space. This transition enables us to consider the discrete field of kinematic and dynamic characteristics of movement as quite close to a continuous field of the same characteristics.

The authors pay special attention to the issue of averaging hydrodynamic quantities characterizing the flow of multiphase media. This is driven by two factors: the peculiar structure of the medium and the turbulent nature of its flow, which necessitated the shift from original structures and quantities to smoother and more regular characteristics that can be studied using conventional methods of mathematical analysis (Ghajar and Bhagwat, 2013).

When determining the average value of the kinematic and dynamic characteristics of two-phase systems, time or spatial averaging over any period of time or region of space is most commonly used. For this purpose, the author also employed a more general method of spatiotemporal averaging in the following form.

Problem statement

Let us consider a multiphase flow with varying mass as a collection of a large number of different groups of particles (molecules, drops, bubbles, solid inclusions, etc.) in continuous chaotic motion. For the purposes of mathematical formulation of such flow, we will employ an averaged motion description introducing the notion of multispeed continuum and interpenetrating motion of its components. A multi-speed continuum consists of a set of individual substances, each belonging to its corresponding component (a phase or component) of the mixture, and occupying a fixed volume within the system. The average density, velocity, temperature and other kinematic and dynamic parameters related to its continuum and its mixture component defined as the functions of the four-dimensional space can be determined for each of these components of continuums within the mixture at each point of the volume.

It is therefore assumed that the elementary mass with velocity $\bar{u}_{*j}$ becomes attached to (or detached from) a mixture particle with the velocity vector $\bar{u}_i$. Here, the velocity vector $\bar{u}_{*i}$ may differ by a certain value from the velocity vector of the main mass of the i phase of $\bar{u}_i$ (i.e. $\bar{u}_i \neq \bar{u}_{*i}$). Since the attached elementary mass may be attached to a fixed particle from different directions, q_* – flow of mass, $\bar{u}_{*i} q_{*i}$ – flow of momentum and $(e_{*i} + u_{*i}^2/2) q_{*i}$ – flow of energy attached to a particle per unit of time and per unit of volume should be considered.

Under these conditions, the laws of conservation (mass, momentum and energy) are written as balance equations that relate the rate of change of the "total quantity" of the corresponding physical value (phase or medium) within a specific volume to the "flow" of this value through the surface that restricts the volume, and the "sources" acting within the volume.

Basic motion equations

Let us select an arbitrary volume V limited by a surface S of a multiphase mixture which is moving with the continuous change of mass (i.e. external heat and mass transfer) for the purposes of mathematical formulation of the laws of conservation at a given point of time t. Subsequently, the following integral equations of mass, momentum and energy balance can be written for the i phase of a multiphase medium:

Mass balance equation:

$$\int_V \frac{\partial}{\partial t}(\rho_i \phi_i) = -\int_S (\rho_i \phi_i u_{in}) dS + \int_V (q_{*i} + (-1)^i \chi) dV, \quad i = 1.2 \tag{1}$$

Momentum balance equation:

$$\begin{gathered} \int_V \frac{\partial}{\partial t}(\rho_i \phi_i \vec{u}_i) dV = -\int_S \left[(\rho_i \phi_i \vec{u}_i) u_{in} - \phi_i \sigma_{in} \right] dS + \\ + \int_V \left[(\rho_i \phi_i \vec{F}_i + \vec{u}_{*i} q_{*i}) + (-1)^i (\vec{R}_i + \vec{u}_\chi \chi) \right] dV \end{gathered} \tag{2}$$

Total energy balance equation:

$$\begin{gathered} \int_V \frac{\partial}{\partial t}(e_i + u_i^2) \rho_i \phi_i dV = \\ = -\int_S \left[\rho_i \phi_i (e_i + u_i^2/2) u_{in} - \phi_i \vec{u}_i \sigma_{in} + \phi_i \vec{q}_{in}^{\,*} \right] dS + \\ + \int_V \left[\rho_i \phi_i \vec{F}_i \vec{u}_i + (e_{*i} + u_{*i}^2/2) q_{*i} + \right. \\ \left. + (-1)^i \left(\vec{R}_i \vec{u}_i + Q_i + (e_{\chi i} + u_\chi^2/2) \right) \chi \right] dV \end{gathered} \tag{3}$$

In these equations: ρ_i, ϕ_i, u_i are true density, volume concentration and velocity of the i phase; u_{*i} – velocity of attached (or detached) mass; q_{*i} – specific attached (or detached, with $q_{*i} < 0$) mass; χ – specific mass of phase transition; F_i, σ_i – specific vector of mass and stress tensor of superficial forces; u_χ – velocity of interphase transition mass; $\vec{R}_i$ – specific vector of interphase forces; $\vec{r}_i$ – radius vector; n – outward normal; e_i – specific internal energy of the i-phase; e_{*i}, e_χ – specific internal energy of attached (or detached) mass and phase transformations respectively; Q_i – intensity of heat exchange between phases; q_i^* – vector of specific heat flow to the i phase of the mixture.

In the area of continuous motions, the integral equations of mass, momentum and energy balance written for i (carrier or carried) phase (1)–(3), are equivalent to differential equations. If in the first part (1)–(3), integrals taken through a surface S are translated to the integrals taken through a volume V according to the Gauss-Ostrogradsky formula, after the relevant transformations we obtain the following differential equations of mass, momentum and energy transfer for the i phase.

1. Mass transfer equation (equation of continuity):

$$\frac{d}{dt}(\rho_i \phi_i) + \rho_i \phi_i \, div \vec{u}_i = q_{*i} + (-1)^i \chi, \quad i = 1.2 \tag{4}$$

2. Momentum transfer equation:

$$\begin{gathered} \rho_i \phi_i \frac{d\vec{u}_i}{dt} = \rho_i \phi_i \vec{F}_i + div(\phi_i \vec{\sigma}_i) + \\ + (\vec{u}_{*i} - \vec{u}_i) q_{*i} + (-1)^i \left[\vec{R}_i + (\vec{u}_\chi - \vec{u}_i) \chi \right] \end{gathered} \tag{5}$$

3. Total energy transfer equation:

$$\begin{gathered} \rho_i \phi_i \frac{d}{dt} E_i = \rho_i \phi_i \vec{F}_i \vec{u}_i + div\left[\phi(\vec{\sigma}_i \vec{u}_i - \vec{q}_i^{\,*}) \right] + \\ + (E_{*i} - E_i) q_{*i} + (-1)^i \left[\vec{R}_i \vec{u}_i + Q_i + (E_\chi - E_i) \chi \right] \end{gathered} \tag{6}$$

where: $E_i = e_i + \frac{u_i^2}{2}$; $E_\chi = e_\chi + \frac{u_\chi^2}{2}$; $E_{*i} = e_{*i} + \frac{u_{*i}^2}{2}$.

Adding (1)–(3) or (4)–(6) for the flow in whole we get the following differential equations:

- equation of through flow:

$$\frac{d\rho}{dt} + \rho\, div\, \vec{u} = q_* \tag{7}$$

- equation of dynamics:

$$\rho \frac{d\vec{u}}{dt} = \rho\vec{F} + div\vec{\sigma} + (\vec{u}_* - \vec{u})q \tag{8}$$

- equation of total energy:

$$\rho \frac{dE}{dt} = \rho(\vec{F}\cdot\vec{u}) + div(\vec{\sigma}\cdot\vec{u}) - div\vec{q}^{\,*} + (E_* - E)q_* \tag{9}$$

where: $E = e + \frac{u^2}{2}$; $E_* = e_* + \frac{u_*^2}{2}$; $\rho = \sum_i \rho_i \phi_i$; $\bar{u} = \frac{\sum_i \rho_i \phi_i \bar{u}_i}{\rho}$; $\bar{\sigma} = \sum_i \phi_i \bar{\sigma}_i$.

For slow processes (i.e., at medium motion speeds that are significantly lower than sonic velocity), the internal (heat) energy equation obtained from the comparison (8) and (9) can be used. For this purpose, we scalarly multiply both parts (8) by the medium velocity vector and subtract the obtained result from (9), then:

$$\rho \frac{de}{dt} = -div\vec{q}^{\,*} - (e - e_*)q_* \tag{10}$$

Thus, the system of basic equations of flow motion with heat and mass transfer (i.e., with the influence of external sources of mass, momentum and energy) is represented in the following form:

$$\frac{d\rho}{dt} + \rho\, div\, \vec{u} = q_* \tag{11}$$

$$\rho \frac{d\vec{u}}{dt} = \rho\vec{F} + div\vec{\sigma} + (\vec{u}_* - \vec{u})q_* \tag{12}$$

$$\rho \frac{de}{dt} = -div\vec{q}^{\,*} - (e - e_*)q_* \tag{13}$$

In the absence of external sources of mass ($q_* = 0$), momentum $[(\vec{u}_* - \vec{u})q_* = 0]$, and energy $[(e - e_*)q_* = 0]$, the known equations of fluid and gas dynamics can be obtained from this system (Duich and Zaryankin, 1984; Anderson et al., 2012):

$$\frac{d\rho}{dt} + \rho\, div\bar{u} = 0;\ \rho\frac{d\bar{u}}{dt} = \rho\bar{F} + \nabla\bar{\sigma};\ \rho\frac{de}{dt} = -\nabla\vec{q}^{\,*} \tag{14}$$

The obtained equations are valid for describing the motion of a component of the mixture and flow as a whole with any physical properties. However, this system is not truly defined. It is necessary to incorporate thermodynamic and rheological state equations, as well as heat flow equations, into it. These additional relations are established when constructing a mathematical model of the specific environment under study.

As an example, consider the flow of a viscous incompressible medium. These additional relations are established when building a mathematical model of the specific medium under study. As an example, let us consider the flow of a viscous incompressible medium. The condition of incompressibility of the medium applies in cases where the flow velocity of the fluid (gas) is significantly lower than the noise velocity. Apart from substances such as liquid (oil, oil products, water, etc.), which are practically incompressible media, this condition is partially fulfilled for gas (air) flows when the Mach number in them is $M_a \le 0.3$. In such cases, the following additional relations should be written:

a) for stress tensor of superficial forces:

$$\sigma_{ij} = -p\sigma_{ij} + \tau_{ij} \tag{15}$$

where: p – pressure; σ_{ij} – Kronecker symbol; τ_{ij} – viscous tension tensor (in incompressible Newtonian media); $\tau_{ij} = \mu\varepsilon_{ij}$; ε_{ij} – strain velocity tensor, $\varepsilon_{ij} = \partial u_i / \partial x_j + \partial u_j / \partial x_i$; μ – coefficient of dynamic viscosity;

b) for heat flows (according to the Fourier's law):

$$q^* = -\lambda\nabla T \tag{16}$$

where: λ – heat-conduction coefficient.

c) for internal energy:

$$e = cT,\ e_* = cT_* \tag{17}$$

where: T, T_* – temperatures of the main and attached (or detached) mass of the medium; c – specific heat capability (in incompressible media $c = c_p = c_V$).

By adding expressions (15)–(17) to the system (11)–(13), we obtain hydrothermodynamic equations for a viscous incompressible medium with heat and mass transfer:

$$\nabla\bar{u} = q \tag{18}$$

$$\frac{\partial\bar{u}}{\partial t} + (\bar{u}\cdot\nabla)\bar{u} = \bar{F} - \rho^{-1}\nabla P + \nu\nabla^2\bar{u} + (\bar{u}_* - \bar{u})q \tag{19}$$

$$\frac{\partial T}{\partial t} + (\bar{u}\cdot\nabla)T = a\nabla^2 T + (T_* - T)q \tag{20}$$

where: $\nu = \mu/\rho$; $a = \lambda/\rho c$; $q = q_*/\rho$.

The known Navier-Stokes motion equations arise as a special case from (18)–(20) if there is no influence of external sources (or flow-offs) of mass $q = 0$, momentum $(\bar{u}_* - \bar{u})q = 0$ and energy $(T_* - T)q = 0$ (Duich and Zaryankin, 1984):

$$\nabla\bar{u} = 0,\ \frac{\partial\bar{u}}{\partial t} + (\bar{u}\cdot\nabla)\bar{u} = \bar{F} - \rho^{-1}\nabla P + \nu\nabla^2\bar{u} \tag{21}$$

and the heat transfer equation in a viscous incompressible medium:

$$\frac{\partial T}{\partial t} + (\bar{u}\cdot\nabla)T = a\nabla^2 T \tag{22}$$

Therefore, the established equations (18)–(21) are more universal and applicable to a wide range of thermophysical and hydrodynamic problems.

Analysis of these systems of equations shows that they form a closed system of five equations to find $\bar{u}(u_x, u_y, u_z)$, P and T (values q, v, ρ, c, a, T_*, u_*, F in this system are predetermined). When solving specific problems, the conditions of unambiguity (boundary conditions) including geometric, physical, time (initial) and boundary conditions should be added to the system (18)–(20).

Thus, the system of differential equations (18)–(20) along with the conditions of unambiguity represents a mathematical statement of boundary value problems for macromedia with heat and mass transfer.

The problem is solved by analytical, numerical or experimental methods. In the latter case, widely used methods of physical and mathematical modelling can be used.

It is obvious that the exact analytical solution of the boundary value problem (described by the system of differential equations of motion of the macro medium with heat and mass transfer) is difficult in general terms. Under these conditions, it is necessary to use numerical methods.

Similarity criteria

The analytical solution of the boundary value problem of hydromechanical processes with varying mass is extremely challenging to tackle mathematically. Due to the introduction of additional terms (which take into account the mechanical and thermal interactions that significantly complicate the boundary conditions) into the motion equations, a purely analytical study of these processes is now possible only with an approximate problem statement. This allows for a certain simplification of the original equations by either neglecting terms that are not essential for this problem or by replacing complex exact relations between approximate values. This substitution in the study of complex systems is known as a similarity theory. It is a crucial tool for modelling hydrodynamic processes in heat-conducting flow given their complex nature.

Below, we discuss the conditions for achieving similarity in hydrodynamic processes in macroscale with external heat and mass exchange. The influence of individual terms in motion equations is evaluated either through experimental verification or numerical methods. Generalization and extension of these data to similar phenomena are more straightforward when transitioning from ordinary physical values to complex values composed in a specific way depending on the nature of the process. In this case, the number of variables decreases and the interrelationships characterizing the phenomenon as a whole become more pronounced. This replacement of ordinary variables with generalized ones in the study of complex systems is called similarity theory. One of the key challenges in this theory is establishing rules for making generalizations and extending the results of experiments conducted under certain conditions to other conditions, as well as determining the limits of the applicability of these generalizations. It is evident that similarity theory is a crucial tool for analysing hydrodynamic processes in heat-conductive media given their complex nature (Sedov, 1977; Anderson et al., 2012).

To ensure the similarity of the simulated flows, some dimensionless groups, known as similarity numbers, must be equal. These numbers can be determined in two ways: either derived from the equations of motion of the process or identified through dimensional analysis. Moreover, the tools of the first method are somewhat simpler compared to those of dimensional analysis (Cou, 1971; Kryukov, 2003).

Let us proceed to the consideration of the similarity conditions of two flows of viscous incompressible media with variable mass. The conditions for achieving hydrodynamic and thermal similarity in these are obtained by writing the non-dimensional motion equations and equating the numerical coefficients in both systems. To derive the corresponding similarity criteria, we present a system of motion equations (continuity, dynamics and energy) with varying mass (18)–(20) in the following form (here and elsewhere, the dynamic equations are carried out only in the projection onto X axis):

$$\begin{cases} \rho\left(\dfrac{\partial u}{\partial x}+\dfrac{\partial \upsilon}{\partial y}+\dfrac{\partial w}{\partial z}\right)=q_* \\ \rho\left(\dfrac{\partial u}{\partial t}+u\dfrac{\partial u}{\partial x}+\upsilon\dfrac{\partial u}{\partial y}+w\dfrac{\partial u}{\partial z}\right)= \\ =\rho F_x-\dfrac{\partial P}{\partial x}+\mu\left(\dfrac{\partial^2 u}{\partial x^2}+\dfrac{\partial^2 u}{\partial y^2}+\dfrac{\partial^2 u}{\partial z^2}\right)+\left(u_*-u\right)q_* \\ \rho C\left(\dfrac{\partial T}{\partial t}+u\dfrac{\partial T}{\partial x}+\upsilon\dfrac{\partial T}{\partial y}+w\dfrac{\partial T}{\partial z}\right)= \\ =\lambda\left(\dfrac{\partial^2 T}{\partial x^2}+\dfrac{\partial^2 T}{\partial y^2}+\dfrac{\partial^2 T}{\partial z^2}\right)+C\left(T_*-T\right)q_* \end{cases} \tag{23}$$

where: ρ – density; u, υ, w – projections of medium velocity vector onto axes of reference x, y, z; P – pressure; μ – dynamic (shearing) viscosity; T – medium temperature; C – heat capacity; λ – heat-conduction coefficient; T_*, q_* – temperature and specific attached (or detached, with $q_* < 0$) mass of medium.

Let us reduce the equations (23) to a dimensionless form by using scales for time, length (particularly, coordinates), velocities, pressures, mass forces, etc.

For this purpose, let us denote the dimensionless values with the same letters as the dimensional values but with a dash, and make the following substitution:

$$\begin{cases} t = t_0\bar{t}, \quad x = l_0\bar{x}, \quad y = l_0\bar{y}, \quad z = l_0\bar{z}, \\ u = V_0\bar{u}, \quad \upsilon = V_0\bar{\upsilon}, \quad w = V_0\bar{w}, \quad P = P_0\bar{P}, \\ T = T_0\bar{T}, \quad T_* = T_{*0}\bar{T}_*, \quad q_* = q_{*0}\bar{q}_*, \\ F_x = g\bar{F}_x, \quad F_y = g\bar{F}_y, \quad F_z = g\bar{F}_z \end{cases} \tag{24}$$

Substituting these values $t, x, ..., P, ...$ in the equations system (23), we get:

$$\begin{cases} \dfrac{\rho V_0}{l_0}\left(\dfrac{\partial \bar{u}}{\partial \bar{x}} + \dfrac{\partial \bar{\upsilon}}{\partial \bar{y}} + \dfrac{\partial \bar{w}}{\partial \bar{z}}\right) = q_{*0}\,\bar{q}_* \\ \dfrac{\rho V_0}{t_0}\dfrac{\partial \bar{u}}{\partial \bar{t}} + \dfrac{\rho V_0^2}{l_0}\left(\bar{u}\dfrac{\partial \bar{u}}{\partial \bar{x}} + \bar{\upsilon}\dfrac{\partial \bar{u}}{\partial \bar{y}} + \bar{w}\dfrac{\partial \bar{u}}{\partial \bar{z}}\right) = \\ = \rho g \bar{F}_x - \dfrac{P_0}{l_0}\dfrac{\partial \bar{P}}{\partial \bar{x}} + \dfrac{\mu V_0}{l_0^2}\left(\dfrac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \dfrac{\partial^2 \bar{u}}{\partial \bar{y}^2} + \dfrac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right) + \\ + \left(V_{*0}\,\bar{u}_* - V_0\,\bar{u}\right) q_{*0}\,\bar{q}_* \\ \dfrac{\rho C T_0}{t_0}\dfrac{\partial \bar{T}}{\partial \bar{t}} + \dfrac{\rho C V_0 T_0}{l_0}\left(\bar{u}\dfrac{\partial \bar{T}}{\partial \bar{x}} + \bar{\upsilon}\dfrac{\partial \bar{T}}{\partial \bar{y}} + \bar{w}\dfrac{\partial \bar{T}}{\partial \bar{z}}\right) = \\ = \dfrac{\lambda T_0}{l^2}\left(\dfrac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \dfrac{\partial^2 \bar{T}}{\partial \bar{y}^2} + \dfrac{\partial^2 \bar{T}}{\partial \bar{z}^2}\right) + C\left(T_{*0}\,\bar{T}_* - T_0\,\bar{T}\right) q_{*0}\,\bar{q}_* \end{cases} \tag{25}$$

then, by reducing both parts of these equations to a suitably chosen combination of scales and physical constants, let us minimize the number of complexes within the equations. Let us divide both parts of the continuity equation by $(\rho V_0)/l_0$, the dynamic equation by $(\rho V_0^2)/l_0$, and the energy equation by $(\rho C V_0 T_0)/l_0$ and omit the dashes over the dimensionless values for simplicity, resulting in (26).

From this system, it can be concluded that if two media flows are similar, since they are described by identical equations (as well as with the similar boundary conditions):

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial \upsilon}{\partial y} + \dfrac{\partial w}{\partial z} = J_* q_* \\ Sh\dfrac{\partial u}{\partial t} + u\dfrac{\partial u}{\partial x} + \upsilon\dfrac{\partial u}{\partial y} + w\dfrac{\partial u}{\partial z} = \dfrac{1}{Fr}F_x - Eu\dfrac{\partial P}{\partial x} + \\ + \dfrac{1}{\mathrm{Re}}\left(\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} + \dfrac{\partial^2 u}{\partial z^2}\right) + \left(k_* u_* - u\right) J_* q_* \\ Sh\dfrac{\partial T}{\partial t} + u\dfrac{\partial T}{\partial x} + \upsilon\dfrac{\partial T}{\partial y} + w\dfrac{\partial T}{\partial z} = \\ = \dfrac{1}{Pe}\left(\dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}\right) + \left(\theta_* T_* - T\right) J_* q_* \end{cases} \tag{26}$$

presented in a dimensionless form, then the following dimensionless parameters must be identical for them:

$$\begin{cases} \dfrac{l_0}{V_0 t_0} = Sh; \quad \dfrac{V_0^2}{g\,l_0} = Fr; \quad \dfrac{P_0}{\rho V_0} = Eu; \quad \dfrac{V_0\,l_0}{\upsilon} = \mathrm{Re}; \\ \dfrac{V_0\,l_0}{a} = Pe; \quad \dfrac{q_{*0}\,l_0}{V_0} = J_*; \quad \dfrac{V_{*0}}{V_0} = k_*; \quad \dfrac{T_{*0}}{T_0} = \theta_* \end{cases} \tag{27}$$

where: $a = \lambda/\rho C$ – temperature conductivity coefficient.

Thus, for such physical phenomena (hydrodynamic, thermal), the basic system of motion equations, written in dimensionless form (26), must be the same. For such phenomena, the numbers Sh, Fr, Eu, Re, Pe, k_*, θ_*, J_* defined by established parameters should be similar (the condition of identity of similarity numbers is denoted by a symbol (*idem*) which replaces the phrase "the same value") because these numbers are also known as similarity criteria, i.e.

$Sh = idem$; $Fr = idem$; $Eu = \text{const}$; $Re = idem$;
$Pe = idem$; $J_* = idem$; $k_* = idem$; $\theta_* = idem$.

The physical meaning of similarity numbers Sh, Fr, Eu, Re, Pe, k_*, θ_*, J_* can be established after considering the physical content of each term in the motion equation.

An analysis of the equations of motion of the medium with heat and mass transfer that the Strouhal number Sh expresses the ratio of the local inertial force to the convective one; the Froude number Fr characterizes the ratio of inertia to gravity; the Euler number Eu characterizes the ratio of the force of pressure to the force of inertia; the Reynolds number Re expresses the ratio of the force of inertia to the force of viscosity; the Peclet number Pe characterizes the convection and heat-conducting heat transfer in flowing media; the new numbers J_*, k_*, θ_* characterize a measure of the ratio between the mass flow rate, velocity and temperature of the added (attached or detached) and main medium flows. These numbers characterize the ratio of the values of different physical nature and serve as similarity criteria.

Usually, in cases where any physical value included in the similarity criterion cannot be determined experimentally or calculated, it is excluded by rearranging two or more similarity criteria while obtaining the so-called derived similarity criteria. They can be obtained by combining the basic numbers of thermal and hydrodynamic similarity. For example, determining the flow rate with free movement (natural convection) is particularly challenging due to variations in the density of the medium caused by temperature differences at its various points. This can be addressed by combining the Reynolds Re and Froude Fe numbers. The Galileum number $Ga = Re^2/Fr$, which reflects the influence of the gravity field represented by free-fall acceleration on processes occurring in a medium of this viscosity, can be derived from the ratio of Re and Fr.

The nomenclature and number of similarity criteria are selected depending on the problem at hand. It is possible that self-similar areas may emerge in which the identification of a particular criterion becomes degenerate. In a number of practical problems, approximate similarity can be considered when the number of characteristic criteria is reduced to the minimum, which can be implemented in a model or numerical experiment.

Conclusion

Non-Newtonian viscous fluids are commonly encountered in nature and find extensive applications in various fields of technology, especially in the oil, gas and chemical industries.

The primary characteristic of non-Newtonian viscous fluids is their flow curves (rheological curves or rheograms), depicting graphs of the relationship between the velocity gradient (or shear rate) and the resulting shear stresses within the fluid.

Flow curves are constructed on the basis of experimental data obtained through viscometric research.

As is known, Newtonian fluids exhibit linear flow curves. The viscosity of such fluids is determined by the slope of the corresponding direct rheogram to the horizontal axis and is the only constant characterizing the rheological properties of a viscous fluid at a given temperature and pressure, regardless of the velocity gradient.

The flow curves of non-Newtonian fluids are very diverse and, in general, are not linear.

These fluids include power-law, viscoplastic and other non-Newtonian fluids.

The necessary equations for the hydromechanics of multiphase media with varying mass and the similarity criteria for hydrodynamic processes modelling are derived from the laws of conservation of mass, momentum and energy.

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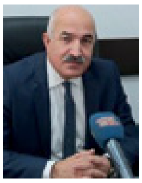

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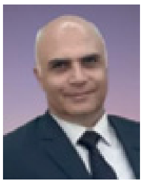

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