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Studying the influence of selected controlled factors on the coefficient of sliding friction through experimental planning methods

Badanie wpływu wybranych kontrolowanych czynników na współczynnik tarcia ślizgowego za pomocą eksperymentalnych metod planowania

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ABSTRACT: The article presents the use of mathematical statistics to determine controllable factors that influence the coefficient of friction in a dual-flow three-stage spur gear transmission of a newly designed sucker rod pumping unit. The load acting on the bearing units, the rotational speed of the main and auxiliary shafts, and the diametrical gap between the shaft and the bushing were assessed through tests. The tests were carried out in random order. The multivariate testing was used to determine the number of tests to be performed as well as the test conditions in order to achieve the required accuracy bushing. In the statistical analysis of the results, the Cochrane criterion was applied to verify the homogeneity of the variance values. Student's *t*-test was used to verify the significance of the coefficients included in the regression equation, Fisher's *F*-test was used to determine the suitability of the adopted output function for describing the real object of study, i.e., the adequacy of the model, and sensitivity coefficients were used to assess the influence of the corresponding parameters on the optimization parameter. The analysis also considered the dispersion of optimization parameters, measurement repeatability, and errors in the model coefficients. Experimental values of the criteria, along with the dispersion of the adequacy and repeatability of the mathematical model, were evaluated to determine whether the model is fully suitable for the object of study. The impact of significant factors and their combinations, as well as their critical values, were assessed by comparing calculated values of the criteria. The significance or insignificance of the corresponding coefficients of the regression equation was determined. To investigate the impact of these controlled factors — load, rotational speed of the main and auxiliary shafts, and the diametrical gap between the auxiliary shaft and the bushing — on the sliding friction coefficient, a modernized device was developed. This device simulates the operating conditions of a double friction sliding bearing in a dual-flow, three-stage spur gearbox bushing. The experiments were conducted using I-40A industrial oil at room temperature. It has been determined that these factors significantly impact the coefficient of friction in a double friction bearing. Consequently, it is necessary to calculate their limit values to ensure the bearing assembly operates without failure during the required service life. Compared to other examined parameters, the optimization parameter, i.e., the coefficient of friction, is most significantly influenced by the diametrical gap between the auxiliary shaft and the bushing, and least affected by the magnitude of the load acting on the bearing assembly. The overall impact of controlled factors on the coefficient of friction is minimal. Employing the test planning method, a mathematical formula was derived, enabling the determination of the coefficient of friction in a double sliding friction bearing without additional tests within the range of limit bushing values of contact parameters.

Key words: pumping unit, transmission, sliding bearing, coefficient of friction, dispersion, regression.

STRESZCZENIE: W artykule przedstawiono zastosowanie statystyki matematycznej do określenia sterowalnych czynników wpływających na współczynnik tarcia w podwójnej, trójstopniowej przekładni zębatej czołowej nowo zaprojektowanej pompy żerdziowej. Za pomocą badań oceniono obciążenie działające na zespoły łożyskowe, prędkość obrotową wału głównego i pomocniczego oraz średnicę szczeliny między wałem a tuleją. Badania przeprowadzono w losowej kolejności. Badania wielowariantowe wykorzystano do określenia liczby badań, które należy wykonać, a także warunków badawczych w celu osiągnięcia wymaganej dokładności. W analizie statystycznej wyników zastosowano kryterium Cochrane'a w celu weryfikacji jednorodności wartości wariancji. Test *t*-Studenta posłużył do weryfikacji istotności współczynników zawartych w równaniu regresji, test *F*-Fishera wykorzystano do określenia przydatności przyjętej funkcji wyjściowej do opisania rzeczywistego obiektu badań, tj. adekwatności modelu, a współczynniki wrażliwości wykorzystano do oceny wpływu odpowiednich parametrów na parametr optymalizacyjny. W analizie uwzględniono również rozrzut parametrów optymalizacyjnych, powtarzalność pomiarów oraz błędy we współczynnikach modelu. Eksperymentalne wartości kryteriów, wraz z rozproszeniem adekwatności i powtarzalności modelu matematycznego, zostały ocenione w celu ustalenia czy model w pełni spełnia wymagania stawiane obiektowi badań. Wpływ istotnych czynników i ich kombinacji, a także ich wartości krytyczne, oceniono

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poprzez porównanie obliczonych wartości kryteriów. Określono istotność lub nieistotność odpowiednich współczynników równania regresji. W celu zbadania wpływu tych kontrolowanych czynników – obciążenia, prędkości obrotowej wału głównego i pomocniczego oraz szczeliny średnicowej pomiędzy wałem pomocniczym a tuleją – na współczynnik tarcia ślizgowego, opracowano zmodernizowane urządzenie. Urządzenie to symuluje warunki pracy podwójnego łożyska ślizgowego w podwójnej, trzystopniowej przekładni czołowej. Eksperymenty przeprowadzono przy użyciu oleju przemysłowego I-40A w temperaturze pokojowej. Ustalono, że czynniki te mają znaczący wpływ na współczynnik tarcia w podwójnym łożysku ślizgowym. W związku z tym konieczne jest obliczenie ich wartości granicznych, aby zapewnić bezawaryjną pracę zespołu łożyskowego w wymaganym okresie eksploatacji. W porównaniu z innymi badanymi parametrami, na parametr optymalizacyjny, tj. współczynnik tarcia, największy wpływ ma szczelina średnicowa między wałem pomocniczym a tuleją, a najmniejszy wpływ ma wielkość obciążenia działającego na zespół łożysk. Ogólny wpływ kontrolowanych czynników na współczynnik tarcia jest minimalny. Wykorzystując metodę planowania badań, wyprowadzono wzór matematyczny umożliwiający wyznaczenie współczynnika tarcia w podwójnym ślizgowym łożysku bez dodatkowych badań w zakresie granicznych wartości parametrów styku.

Słowa kluczowe: pompa żerdziowa, przekładnia, łożysko ślizgowe, współczynnik tarcia, dyspersja, regresja.

Introduction

Oil production and import crucial for the development of many countries. Pumping units are an integral technology in the oil industry. At present, various types of pumping units and installations are employed for mechanized exploitation of oil wells.

Of the existing mechanized methods of oil production, the most common is the sucker-rod pumping machine with balanced individual drives, (Aliverdizade, 1973; Chicherov et al. 1987; Takacs, 2015; Karbage and Costa, 2020). For many years, research has focused on the theory of operation, analysis, and synthesis of the kinematics of both ordinary and unusual balancing of individual drives. This includes the principles of change in force at the rods' point of suspension, pumping unit power calculations, and the design characteristics of typical types of balanced pumping units balanced pumping unit (Ivanovsky et al. 2002; Mishchenko, 2003; Najafov 2013). Their main disadvantages are heavy metal construction , low efficiency, poor balance, the need for a massive foundation, etc. (Aliyeva and Abbasov, 2023).

The high-energy consumption during operation and the consumption of materials during the construction of pumping units has led to the development of new design solutions in this field. One of the main trends in this area is the development of non-beam non-beampumping units. Their advantages are compact dimensions and low metal consumption during construction, lower energy consumption and better dynamic characteristics (Ahmedov et al., 2019; Fakher et al., 2021).

Existing balanced pumping unit technology results in significant fatigue wear of ground equipment due to high loads on the drive. This leads to premature failure and increased energy consumption. These issues necessitates the need for new, improved designs of sucker-rod pumping units. One such innovation is the use of a non-beam sucker-rod pumping units, developed at the Department of Machine Design and Industrial Technologies of Azerbaijan Technical University, as documented in EAPO authorship certificate No. 032268 (Abdullaev et al., 2019).

Figure 1 shows a diagram of a new solution of the non-beam sucker-rod pumping unit. The new solution of the non-beam sucker-rod pump features two cranks (7), rigidly fixed on both sides at the output of the driven shaft of the multi-stage reducer (4), which has only two shafts and a gear ratio of 1:125.

At one end of the reducer's drive shaft, a stepped driven pulley (6) of the V-belt drive (3) is installed, and at the output end, a dual-disk brake (5) is situated.

The mechanical drive consists of a frame (1) constructed from rolled profile forming two longitudinal cross-joints, two brackets for connecting the front pillars (18), two brackets for connecting the rear pillars (19), and two brackets each for connecting the front rods (16) and rear rods (17). Mounted on the frame, along with the reducer, is a three-phase asynchronous electric motor (2).

The transmission also features strings (13, 14) and blocks (11, 12). At one end of the parallel strings, a cross-piece (9) is suspended, to which the rod column (10) is fixed, and at the other end, cranks with a counterweight (8) are attached. On the other end, the traverse is connected to a movable counterweight (15) via parallel strings (14), providing a gain in strength.

The lower end of the rods column (10), rigidly connected to the traverse, is linked to the pump piston. The front and rear drive racks, pivotally connected to the frame brackets, are interconnected by front traction bars on both the right and left sides. In addition, the rear pillars are pivotally connected to the frame brackets using the rear traction rods.

Front and rear traction rods provide a change in the angles of inclination of the front and rear racks. These racks can be telescopic to adjust the height depending on the stroke of the rod suspension. The lower ends (base) of the front and rear pillars are attached to the bracket on the hinged support with the possibility of deflecting to the right or left with the front rods to free space around the wellhead during repair and to precisely control the movement of the rods suspension point vertically.

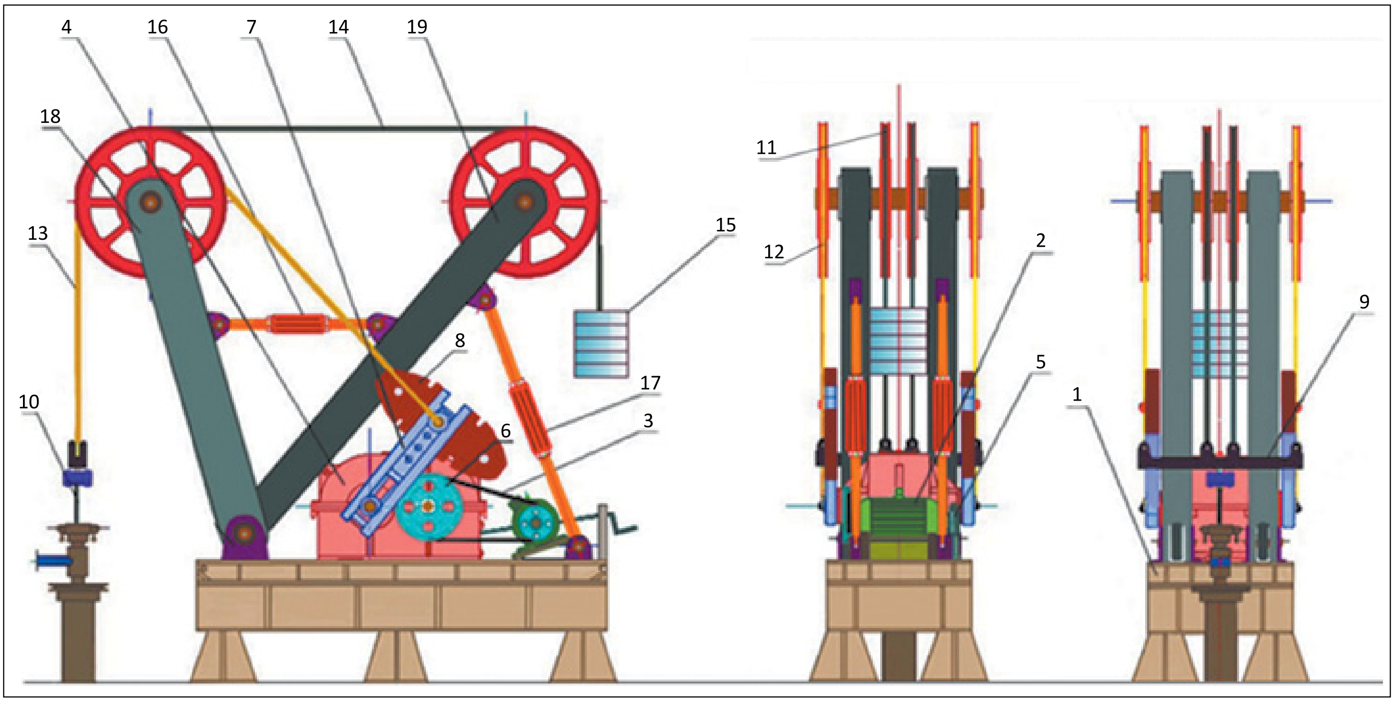

Figure 1. Diagram of a new solution of the non-beam sucker-rod pump unit (see text for explanations)
Rysunek 1. Schemat nowego rozwiązania w zakresie bezramiennych pomp żerdziowych (objaśnienia w tekście)

In addition, the connection of the pillars to the bracket on the hinged support makes it possible to completely unfold the front and rear racks of the mechanical drive during transport to the installation site. To reduce the load on the elements of the transmission by means of a flexible link to the traverse, a movable counterweight is additionally secured (Ahmedov and Hajiyev, 2020; Ahmedov et al., 2021).

Pumping units, which are also called pumpjacks, are the most prevalent type of well pumps. These units are designed to provide synchronous upward and downward pump strokes. This movement is provided by the transmission of the pumping unit. Current research and development efforts focusing on new designs for pumping unit transmissions are highly relevant.

Formulation of the problem

Modern pumping unit transmissions are used for reducing speed and increasing torque acting as gear reducers. Gear reducers, used as transmissions, are used in almost all fields of mechanical engineering and their annual production numbers amount to several million per year. As an integral part of modern machines, the improvement and development of new gear reducers is highly relevant. In development of mechanisms with a high gear ratio multi-stage gearboxes are mainly used.

As a result of the research, it was found that in development of multi-stage gear reducers, the geometric dimensions of their structural elements depend on the working criteria, without taking into account the driving factors. This leads to differences in reliability levels and an increase in metal consumption during production of these structural elements, which as a result reduces the technical level of these mechanical systems. Therefore, to create state-of-the-art gear reducers that can be competitive on the market in line with changes in design philosophy, must be considered through a systematic approach.

In conventional multi-stage gear reducers, increasing the number of stages increases the amount of gears, intermediate shafts and bearings.

And, of course, an increase in the number of structural elements of the gear reducer leads to a decrease in its overall efficiency and reliability level, as well as to an increase in its overall dimensions.

To address these problems, the Department of Machine Design and Industrial Technologies at Azerbaijan Technical University developed a single-line and double-line multi-stage gear reducer, certificated by EAPO (Abdullaev and Najafov, 2019). The design of this reducer was based on a newly created and tested principle of developing structural elements. Figure 2 shows a kinematic diagram of the newly designed three-stage, double-line gear reducer, which was used in the transmission of the considered non-beam sucker-rod pumping unit.

Three stage double-line gear reducer consist of the input shaft (1), output shaft (2), driving gear (3), triple gear block (4), double gear blocks (5, 6), driven gears (7, 8), double

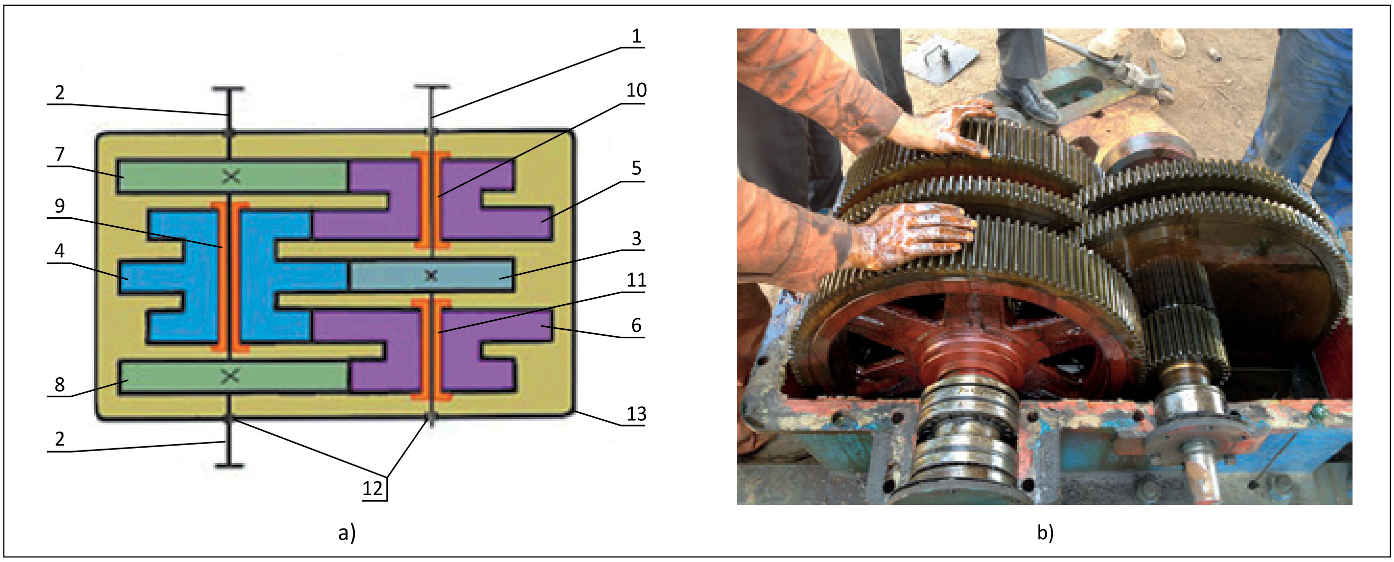

Figure 2. Kinematic diagram (a) and view of the working model (b) of the new designed three-stage double-line gear reducer (see text for explanations)

Rysunek 2. Schemat kinematyczny (a) oraz widok modelu roboczego (b) nowo zaprojektowanego podwójnego trójstopniowego reduktora (objaśnienia w tekście)

sliding bearings (9, 10, 11), rolling-element bearings (12) and reducer housing (13). Driving gear (3) are rigidly fixed to the drive shaft (1) and double gear blocks (5, 6), and freely rotate around the input shaft. Driven gears (7, 8) are rigidly fixed to the output shaft (2), triple gear block (4) and freely rotate around the output shaft. The input and output shafts are installed in the reducer housing (13) using rolling-element bearings (12). Providing gear reducer with double and triple gear blocks placed along the length of the input and output shafts which are freely rotating around them and forming the next gear stages allows reducing the number of intermediate shafts and their bearings.

For structural and economic reasons, double sliding bearings were used for the rotating gear block (4, 5, 6) instead of rolling-element bearings. These sliding bearings were placed under gear blocks (4, 5, 6), mounted on rotating shafts and freely rotate around them.

The estimation of the coefficient of sliding friction of double sliding bearings in a dual-flow, three-stage spur gear reducer being part of the transmission of a newly designed pumping unit, is crucial. It is essential to assess how this coefficient varies depending on controlled factors and to maintain its stability during operation.

Theoretically, ensuring the minimum values of the coefficient of friction of double sliding bearings in the gear reducer contributes to the operation of the bearing assembly in much more optimal conditions and, accordingly, the maximum efficiency with minimal friction losses. Therefore, the assessment of the influence of certain controlled factors on the coefficient of friction is one of the highest priority tasks. The use of mathematical and statistical techniques is of particular importance in this experimental design.

Solution of the problem

The coefficient of sliding friction of double sliding bearings is influenced by numerous factors. To evaluate the influence of these factors, a mathematical statistical method is used.

To study the influence of certain controlled factors, i.e., the load on the bearing assemblies, the rotational speed of the main and auxiliary shafts, and the diametrical gap between the auxiliary shaft and the bushing on the coefficient of sliding friction— experiments were performed on modernized ДМ-29М equipment, certified by The State Committee on Standardization, Metrology, and Patents of the Azerbaijan Republic (Abdullaev et al., 2019). These experiments were conducted at room temperature using I-40A industrial oil. bushing.

To solve the problem with the required accuracy in determining the number of experiments and the conditions for their implementation, the method of planning multi-factor experiments was used (Adler et al. 1976; Zakharova, 2003; Makarichev and Ivannikov, 2016). As four variable factors were adopted in the experiments, to solve the regressive equation of the coefficient of friction, 2k (here k = 4 is the number of controlled factors) and a typical full-factor experiment (Panevnyk, 2021) were assumed. The numerical values of the factors used in the planning are presented in Table 1.

The coded values of the factors adopted are as follows:

$$\begin{cases} X_1 = \dfrac{F - 0.5\left(F_{\max} + F_{\min}\right)}{0.5\left(F_{\max} - F_{\min}\right)};\ X_2 = \dfrac{n_1 - 0.5\left(n_{1\max} + n_{1\min}\right)}{0.5\left(n_{1\max} - n_{1\min}\right)}; \\ X_3 = \dfrac{n_2 - 0.5\left(n_{2\max} + n_{2\min}\right)}{0.5\left(n_{2\max} - n_{2\min}\right)};\ X_4 = \dfrac{\Delta - 0.5\left(\Delta_{\max} + \Delta_{\min}\right)}{0.5\left(\Delta_{\max} - \Delta_{\min}\right)} \end{cases} \tag{1}$$

If the values of the factors provided in Table 1 are taken into account:

$$\begin{cases} X_1 = 0.00067F - 1.6667;\ X_2 = 0.0012n_1 - 1.9268 \\ X_3 = 0.0125n_2 - 3.25;\ X_4 = 21.0526\Delta - 2.37 \end{cases} \tag{2}$$

Table1. The main factors and their characteristics
Tabela 1. Główne czynniki i ich charakterystyka

Degree of variation	Code of factor	Load, F	Rotational speed of main shaft, n_1	Rotational speed of auxiliary shaft, n_2	The gap between the auxiliary shaft and the bushing, Δ
		[N]	[min^{-1}]	[min^{-1}]	[mm]
Upper level	$X_{yi}(+)$	4000	2400	340	0.1600
Lower level	$X_{ai}(-)$	1000	760	180	0.0650
Variation interval	ΔX_i	1500	820	80	0.0475
Main level	X_i	2500	1580	260	0.1125
Designation of factors		X_1	X_2	X_3	X_4

Table 2. Matrix of four-factor experiment
Tabela 2. Macierz eksperymentu czteroczynnikowego

No.	Factors					Combination of factors										
	X_0	X_1	X_2	X_3	X_4	X_1X_2	X_1X_3	X_1X_4	X_2X_3	X_2X_4	X_3X_4	$X_1X_2X_3$	$X_1X_2X_4$	$X_1X_3X_4$	$X_2X_3X_4$	$X_1X_2X_3X4$
1	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+
2	+	–	+	+	+	–	–	–	+	+	+	–	–	–	+	–
3	+	+	–	+	+	–	+	+	–	–	+	–	–	+	–	–
4	+	–	–	+	+	+	–	–	–	–	+	+	+	–	–	+
5	+	+	+	–	+	+	–	+	–	+	–	–	+	–	–	–
6	+	–	+	–	+	–	+	–	–	+	–	+	–	+	–	+
7	+	+	–	–	+	–	–	+	+	–	–	+	–	–	+	+
8	+	–	–	–	+	+	+	–	+	–	–	–	+	+	+	–
9	+	+	+	+	–	+	+	–	+	–	–	+	–	–	–	–
10	+	–	+	+	–	–	–	+	+	–	–	–	+	+	–	+
11	+	+	–	+	–	–	+	–	–	+	–	–	+	–	+	+
12	+	–	–	+	–	+	–	+	–	+	–	+	–	+	+	–
13	+	+	+	–	–	+	–	–	–	–	+	–	–	+	+	+
14	+	–	+	–	–	–	+	+	–	–	+	+	+	–	+	–
15	+	+	–	–	–	–	–	–	+	+	+	+	+	+	–	–
16	+	–	–	–	–	+	+	+	+	+	+	–	–	–	–	+

Table 3. Results of experiments and their mathematical characteristics
Tabela 3. Wyniki eksperymentów i ich matematyczny opis

Experiment No.	Coefficient of friction. *f*			Mean value $\bar{f}_u$	Standard deviation. S_u	Dispersion. S_u^2	$\bar{f}'_u$	$\left(\bar{f}_u - f'_u\right)^2$
	f_1	f_2	f_3					
1	0.03114	0.01956	0.02147	0.02406	0.006208	$3.85 \cdot 10^{-5}$	0.026109	$4.1984 \cdot 10^{-6}$
2	0.04873	0.03924	0.04218	0.04338	0.004858	$2.36 \cdot 10^{-5}$	0.043768	$1.5054 \cdot 10^{-7}$
3	0.06704	0.06347	0.06553	0.06535	0.001792	$0.32 \cdot 10^{-5}$	0.067144	$3.2184 \cdot 10^{-6}$
4	0.03854	0.04128	0.03715	0.03899	0.002101	$0.44 \cdot 10^{-5}$	0.039422	$1.8662 \cdot 10^{-7}$
5	0.03672	0.03846	0.03351	0.03623	0.002511	$0.63 \cdot 10^{-5}$	0.034367	$3.4708 \cdot 10^{-6}$
6	0.07215	0.06342	0.06881	0.06813	0.004404	$1.94 \cdot 10^{-5}$	0.067963	$2.7889 \cdot 10^{-8}$

cont. Table 3/cd. Tabela 3

Experiment No.	Coefficient of friction, f			Mean value $\bar{f}_u$	Standard deviation, S_u	Dispersion, S_u^2	$\bar{f}'_u$	$(\bar{f}_u - f'_u)^2$
	f_1	f_2	f_3					
7	0.04346	0.04973	0.04521	0.04613	0.003235	$1.05 \cdot 10^{-5}$	0.045620	$2.6010 \cdot 10^{-7}$
8	0.05543	0.05781	0.05124	0.05483	0.003326	$1.11 \cdot 10^{-5}$	0.053603	$1.5055 \cdot 10^{-6}$
9	0.03072	0.03118	0.02786	0.02992	0.001798	$0.33 \cdot 10^{-5}$	0.031910	$3.9601 \cdot 10^{-6}$
10	0.08671	0.08314	0.09172	0.08719	0.004310	$1.86 \cdot 10^{-5}$	0.087498	$9.4864 \cdot 10^{-8}$
11	0.07443	0.07138	0.07971	0.07517	0.004214	$1.77 \cdot 10^{-5}$	0.076636	$2.1492 \cdot 10^{-6}$
12	0.08226	0.08554	0.09123	0.08634	0.004538	$2.06 \cdot 10^{-5}$	0.086842	$2.5200 \cdot 10^{-7}$
13	0.04208	0.04861	0.04772	0.04614	0.003541	$1.25 \cdot 10^{-5}$	0.046117	$5.2900 \cdot 10^{-10}$
14	0.06834	0.06274	0.07182	0.06763	0.004581	$2.10 \cdot 10^{-5}$	0.066537	$1.1946 \cdot 10^{-6}$
15	0.07542	0.07973	0.07264	0.07593	0.003572	$1.27 \cdot 10^{-5}$	0.074284	$2.7093 \cdot 10^{-6}$
16	0.06836	0.07082	0.06728	0.06882	0.001814	$0.33 \cdot 10^{-5}$	0.069092	$7.2900 \cdot 10^{-8}$
			Σ			$22.67 \cdot 10^{-5}$		$2.3452 \cdot 10^{-5}$

The matrices of the experimental design are shown in Table 2, and their results and mathematical characteristics in Table 3.

The number of experiments N depending on the number of factors is determined as follows:

$$N = 2^k = 2^4 = 16$$

Given the influence of factors, the mathematical model is composed in the form of a polynomial of a first-order:

$$\begin{aligned} f = {} & b_0 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + \\ & + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{34}X_3X_4 + \\ & + b_{123}X_1X_2X_3 + b_{124}X_1X_2X_4 + b_{134}X_1X_3X_4 + b_{234}X_2X_3X_4 + \\ & + b_{1234}X_1X_2X_3X_4 \end{aligned} \tag{3}$$

where: b_0, b_1, b_2, b_3, b_{12}, b_{13}, b_{14}, b_{23}, b_{24}, b_{34}, b_{123}, b_{124}, b_{134}, b_{234}, b_{1234} – regression coefficients.

Since the number of repeated experiments is $m = 3$, the mean value of the output parameters and the mean square deviation of the experiment u were determined in accordance with the following formula:

$$\bar{f}_u = \frac{1}{3}\sum_{q=1}^{3} f_{uq} \tag{5}$$

where: $u = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16$; $q = 1, 2, 3$

$$S_u = \sqrt{\frac{1}{2}\left[\sum_{q=1}^{3} f_{uq}^2 - \frac{1}{3}\left(\sum_{q=1}^{3} f_{uq}\right)^2\right]} \tag{6}$$

where: f_{uq} – obtained value of the coefficient of friction in the repetition of q experiment u. For excample:

$$S_{u1} = \sqrt{\frac{1}{2}\left[(0.03114^2 + 0.01956^2 + 0.02147^2) - \frac{1}{3}(0.03114 + 0.01956 + 0.02147)^2\right]} = 0.006208$$

According to the test results, the coefficients of the output function are calculated based on the following formula:

$$b_i = \frac{1}{N}\sum_{u=1}^{N} X_{iu}\bar{f}_u \tag{7}$$

For excample, influence of b_1 coefficient on factor X_1

$$b_1 = \frac{1}{N}\sum_{u=1}^{N} X_{iu}\bar{f}_u = \frac{1}{16}(0.02406 - 0.04338 + 0.06535 -$$
$$- 0.03899 + 0.03623 - 0.06813 + 0.04613 + 0.05483 +$$
$$+ 0.02992 - 0.08719 + 0.07517 + 0.08634 + 0.04614 -$$
$$- 0.06763 + 0.07517 - 0.06882) = - 0.00727$$

Then

$b_0 = 0.05714$	$b_{13} = -0.00040125$	$b_{124} = 0.00036375$
$b_1 = 0.00727$	$b_{14} = 0.00307875$	$b_{134} = -0.00635625$
$b_2 = -0.006805$	$b_{23} = -0.0033575$	$b_{234} = -0.00168$
$b_3 = -0.00084$	$b_{24} = 0.0026175$	$b_{1234} = -0.00031125$
$b_4 = -0.0100025$	$b_{34} = -0.0033525$	
$b_{12} = -0.00897375$	$b_{123} = -0.00249875$	

Statistical analysis of the results

Verification of the dispersion uniformity

To estimate the variance, the following formula was used:

$$S_u^2 = \frac{1}{m-1}\sum_{q=1}^{3}\left(\bar{f}_u - f_{uq}\right)^2 \tag{8}$$

where:
$m = 3$ – repetition value,
$q = 1, 2, 3$ – repetition number,
$u = 1, 2, \ldots, N$ – row number in the planning matrix,
$N = 16$ – row value in the planning matrix,
f_{uq} – corresponding result obtained in each test,
$\bar{f}_u$ – mean value of the results obtained during the same test.

The results of the calculations are given in Table 3.

$$S_u^2 = \frac{1}{3-1}(0.02406-0.03114)^2 +$$
$$+(0.02406-0.01956)^2 + (0.02406-0.02147)^2 =$$
$$= 3.8542 \cdot 10^{-5}$$

To verify the results of the dispersion uniformity, the Cochran criterion was used:

$$G_h = \frac{S_{u\max}^2}{\sum_{u=1}^{N} S_u^2} \tag{9}$$

where:

$S_{u\max}^2$ – the maximum value of the dispersions calculated based on the results of the experiments,

$\sum_{u=1}^{N} S_u^2$ – sum of the dispersions.

The approximate value of the Cochrane criterion was determined by selecting the corresponding values from Table 3 and substituting them in (9):

$$G_h = \frac{3.95 \cdot 10^{-5}}{22.67 \cdot 10^{-5}} = 0.1698$$

The tests were performed in a random sequence. As noted, the assumed total number of tests was $N = 2^k = 2^4 = 16$ depending on the number of factors N. The procedure for performing the tests was followed by the random numbers shown in the Table 2, 15, 9, 5, 12, 14, 8, 13, 16, 1, 3, 7, 4, 6, 11, 10. Test numbers greater than 8 correspond to repeated experiments. For example, Tests 1 and 9 are identical, thus correspond to Test 1 in the planning matrix. Therefore, as in the table, the Cochrane criterion is assumed to be based on $N = 8$.

From Table 1 (Panevnyk, 2021) the assumed critical value of the Cochrane criterion was $G_b = 0.5157$ based on the significance level $k_1 = m - 1 = 3 - 1 = 2$; $k_2 = N = 8$ and $q = 5\%$ for the number of degrees of freedom. As can be seen, $G_b = 0.1698 < G_b = 0.5157$. Therefore, the condition of homogenous dispersion is met.

Verifying integer significance

Student's t-test was used to assess the impact of the bi coefficients included in the regression equation on their optimization parameters, and their significance.

$$t_i = \frac{|b_i|}{\sqrt{S_b^2}} \tag{10}$$

In this case, the variance of the optimization parameters or the variance of the repetition of values and errors of the model coefficients were calculated as follows:

$$S_f^2 = \frac{1}{N}\sum_{u=1}^{N} S_u^2 = \frac{1}{16} \cdot 22.67 \cdot 10^{-5} = 1.41688 \cdot 10^{-5}$$

$$S_b^2 = \frac{S_f^2}{m \cdot N} = \frac{1.41688 \cdot 10^{-5}}{3 \cdot 16} = 0.029518 \cdot 10^{-5}$$

The experimental values of the Student's t-test for b_0, b_1, b_2, b_3, b_4, b_{12}, b_{13}, b_{14}, b_{23}, b_{24}, b_{34}, b_{123}, b_{124}, b_{134}, b_{234} and b_{1234} coefficients were calculated based on (7):

$$t_0 = \frac{|b_0|}{\sqrt{S_b^2}} = \frac{0.05714}{\sqrt{0.029518 \cdot 10^{-5}}} = 105.17$$

$t_1 = 13.39$, $t_2 = 12.52$, $t_3 = 1.54$, $t_4 = 18.41$, $t_{12} = 16.52$, $t_{13} = 0.7385$, $t_{14} = 5.67$, $t_{23} = 6.18$, $t_{24} = 4.82$, $t_{34} = 6.17$, $t_{123} = 4.60$, $t_{124} = 0.6695$, $t_{134} = 11.70$, $t_{234} = 3.09$, $t_{1234} = 0.5728$.

Based on Table 3 (Chicherov et al., 1987), the critical value of the Student's t-test was assumed to be $t_b = 2.448$ for level of significance $k = (m - 1) \cdot N = (3 - 1) \cdot 16 = 32$ and $q = 5\%$ for the number of degrees of freedom.

If the calculated value of the Student's t-test is greater than the critical value, then the corresponding regression coefficient of the equation is significant, if less, then insignificant and the term in which it enters is derived from the equation. When comparing the computational values of the above-mentioned coefficients to the Student's t-test values with the critical value selected from the table, it appears that the coefficients t_3, t_{13}, t_{124} and t_{1234} were lower than the critical value ($t_3 = 1.54 < t_b = 2.448$; $t_{13} = 0.7385 < t_b = 2.448$; $t_{124} = 0.6695 < t_b = 2.448$; $t_{1234} = 0.5728 < t_b = 2.448$). This means that the coefficients t_3, t_{13}, t_{124} and t_{1234} have a little effect on the output parameter, thus these coefficients cannot be taken into account in the regression equation as non-significant coefficients. Then the above-mentioned regression equation is follows:

$$f = b_0 + b_1X_1 + b_2X_2 + b_4X_4 + b_{12}X_1X_2 + b_{14}X_1X_4 + b_{23}X_2X_3 + $$
$$+ b_{24}X_2X_4 + b_{34}X_3X_4 + b_{123}X_1X_2X_3 + b_{134}X_1X_3X_4 +$$
$$b_{234}X_2X_3X_4 \tag{11}$$

Substituting the values of b_0, b_1, b_2, b_3, b_{12}, b_{13}, b_{23} calculated on the basis of equation (7), in (11) the following is obtained:

$$f' = 0.05714 - 0.00727X_1 - 0.0068X_2 - 0.01X_4 -$$
$$- 0.00897X_1X_2 + 0.0031X_1X_4 - 0.0033X_2X_3 + 0.0026X_2X_4 -$$
$$- 0.0033X_3X_4 - 0.0025X_1X_2X_3 + 0.0064X_1X_3X_4 -$$
$$- 0.00168X_2X_3X_4 \tag{12}$$

Figure 3 shows a diagram of the influence of factors on the coefficients of friction for a more accurate analysis of the resulting mathematical model. As can be seen from the figure, the coefficient of friction of the double sliding bearing is most influenced by the value of X_4, that is, the diametrical gap between the auxiliary shaft and the bushing. The analysis of the two- and three-parameter factors shows that the numerical values of X_1X_4, X_2X_4 and $X_1X_3X_4$ factors must be reduced to reduce the coefficient of friction of the double sliding bearing.

Taking into account the coded equations (2) of factors (12), after some simplification, the dependence of friction coefficients, load, main and auxiliary shafts on the rotational speed, and the diametrical gap between auxiliary shaft and bushing is:

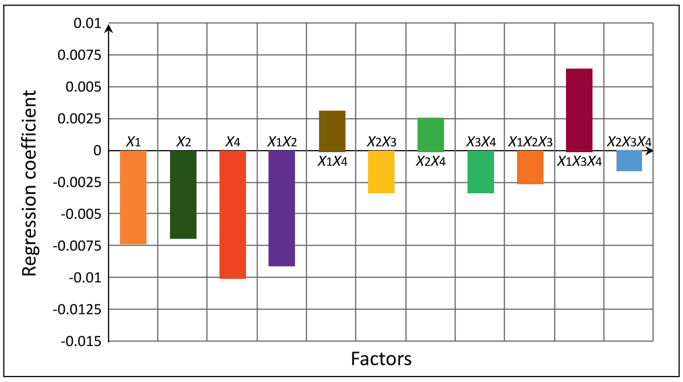

Figure 3. Impact of significant factors and their combinations: negative value –coefficient of friction increases with factor reduction; positive value –coefficient of friction increases with factor increase

Rysunek 3. Wpływ istotnych czynników i ich kombinacji: wartość ujemna – współczynnik tarcia wzrasta wraz z redukcją czynnika; wartość dodatnia – współczynnik tarcia wzrasta wraz ze wzrostem czynnika

$$\begin{aligned} f' = {} & 0.023936 - 1.634\cdot 10^{-5} n_1 + 0.308\cdot\Delta + 0.000297\cdot n_2 + \\ & + 2.414\cdot 10^{-5} F - 0.00282\cdot\Delta n_2 + 0.000248\cdot F\Delta + \\ & + 7.1831\cdot 10^{-8} n_1 n_2 - 8.5841\cdot 10^{-8} F n^2 + 0.0002\cdot n_1\Delta - \\ & - 6.8566\cdot 10^{-10} n_1 F - 5.3053\cdot 10^{-7} n_1 n_2 \Delta - \\ & - 2.5112\cdot 10^{-11} F n_1 n_2 - 1.1207\cdot 10^{-6} F n_2 \Delta \end{aligned} \tag{13}$$

This formula allows to determine the coefficient of friction without additional tests.

Checking the adequacy of the model

To check the output function for description of the actual object of study Fisher's F-test was used:

$$F = \frac{S_{ad}^2}{S_t^2} \tag{14}$$

where: S_{ad}^2 and S_t^2 – dispersion adequacy and repeatability of a mathematical model, respectively.

$$S_{ad}^2 = \frac{1}{N-n}\sum_{u=1}^{N}\left(\bar{f}_u - f'_u\right)^2 \tag{15}$$

$$S_t^2 = \frac{S_f^2}{m} \tag{16}$$

where:

$n = 14$ – number of coefficients in the regression equation,

f'_u –value of the coefficient of friction calculated using the regression equation according to the planning matrix (11) and provided in Table 3.

Using this table, previous value repeating the dispersion is found:

$$S_{ad}^2 = \frac{1}{16-14}\cdot 2.3452\cdot 10^{-5} = 1.1725\cdot 10^{-5}$$

and

$$S_t^2 = \frac{1.41688\cdot 10^{-5}}{3} = 0.47229\cdot 10^{-5}$$

In accordance with (14), the F-test is calculated as:

$$F_h = \frac{1.1725\cdot 10^{-5}}{0.47229\cdot 10^{-5}} = 2.44826$$

Based on Table 4 (Chicherov et al., 1987) the critical value of the F-test value is assumed to be $F_b = 3.98$ based on level of significance $k_1 = N - n = 16 - 14 = 2$; $k_2 = N - (X_{fak} + 1) = = 16 - (4 + 1) = 11$ and $q = 5\%$ for the number of degrees of freedom. As can be seen, $F_h = 2.44826 < F_b = 3.98$. Therefore, the condition of homogenous dispersion is met.

Assessment of the influence of parameters

To assess the influence of the corresponding parameters on the optimization parameter, the sensitivity coefficients are determined based on the following equation:

$$A_i = \frac{|b_i|}{\Delta x_i} \tag{17}$$

then:

$$A_1 = \frac{0.00727}{1500} = 4.84\cdot 10^{-6}$$

$$A_2 = \frac{0.0068}{820} = 8.29\cdot 10^{-6}$$

$$A_4 = \frac{0.01}{0.0475} = 0.2105$$

$$A_{12} = \frac{0.00897}{1500\cdot 820} = 7.2927\cdot 10^{-9}$$

$$A_{14} = \frac{0.0031}{1500\cdot 0.0475} = 0.43509\cdot 10^{-6}$$

$$A_{23} = \frac{0.0033}{820\cdot 80} = 5.0305\cdot 10^{-8}$$

$$A_{24} = \frac{0.0026}{820\cdot 0.0475} = 6.6752\cdot 10^{-5}$$

$$A_{34} = \frac{0.0033}{80\cdot 0.0475} = 8.6842\cdot 10^{-4}$$

$$A_{123} = \frac{0.0025}{1500\cdot 820\cdot 80} = 2.5406\cdot 10^{-11}$$

$$A_{134} = \frac{0.0064}{1500\cdot 80\cdot 0.0475} = 1.1228\cdot 10^{-6}$$

$$A_{234} = \frac{0.00168}{820\cdot 80\cdot 0.0475} = 5.3915\cdot 10^{-7}$$

It is clear that the optimization parameter is most influenced by X_4, that is, the diametrical gap between the auxiliary shaft and the bushing, and the least by factor X_1, that is, the load on the bearings. The combined influence of controlled factors on the coefficient of friction is negligible. Therefore, extreme values of this parameter for the unobtrusive operation of double bearings during operation is to be expected.

Results

The article assesses the effect of load, rotational speed of the main and auxiliary shafts and diametrical gap between the auxiliary shaft and the bushing in the double sliding bearings of the double-line three-stage spur gear reducer in the transmission of a newly constructed non-beam pumping unit. Based on the results of the experiments, it was found that the coefficient of friction on double sliding bearings has a significant impact on the load frequency, the main and auxiliary shaft rotational speed, and the diametrical gap between the auxiliary shaft and the bushing. Compared with the other parameters considered, the coefficient of friction is most affected by the diametrical gap between the auxiliary shaft and the bushing, and the least by the load effect on the bearings.

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